Closing Tues: HW 9.7(2), 9.8, 9.9
Exam 1 is Thur, Jan. $25^{\text {th }}$ covers 9.3-9.9.

### 9.9 Applications (continued)

Recall from last class (and Math 111):
If $p(x)=$ selling price per item (demand)
$A C(x)=$ average cost per item
then

$$
T R(x)=p(x) \cdot x
$$

$$
T C(x)=A C(x) \cdot x .
$$

Entry Task (directly from HW 9.9/5) The price of a certain product is $\$ 400$.
The cost per unit of producing the product is $130+0.5 x$ dollars/item.

Give the functions $T R(x), T C(x), M R(x)$ and $M C(x)$.

Recall from Math 111:
Profit and marginal profit are given by

$$
\begin{aligned}
P(x) & =T R(x)-T C(x) \\
M P(x) & =M R(x)-M C(x)
\end{aligned}
$$

When profit is maximized

$$
M R(x)=M C(x)
$$

Specifically, where it switches from $M R>M C$ to $M R<M C$.

Continuing example from the entry task: How many units should the firm produce and sell to maximize its profits?

Another example
(directly from an old midterm):

You sell items.
If $q$ is in hundred items, then $\operatorname{TR}(q)$ and $\mathrm{TC}(\mathrm{q})$ in hundred dollars are given by

$$
\begin{aligned}
& T R(q)=30 q \\
& T C(q)=q^{3}-15 q^{2}+78 q+10
\end{aligned}
$$

a. Find marginal cost at 2 hundred items
c. What is the maximum value of profit?

Graphs and Derivatives
Example: Let $f(x)=2 x^{2}-3 x$
Find $f^{\prime}(x)$.

$$
f(x)=2 x^{2}-3 x
$$


$f^{\prime}(x)=4 x-3$


Notes/Observations: Given $y=f(x)$.

- $y=f^{\prime}(x)$ is a new function.
- $f(x)=$ "height of the graph at $x$ "
- $f^{\prime}(x)=$ "slope of $f(x)$ at $x^{\prime \prime}$
- $f^{\prime}(x)$ is "instantaneous rate of change" (speedometer speed)
- The units of $f^{\prime}(x)$ are $\frac{\mathrm{y} \text {-units }}{\mathrm{x} \text {-units }}$.


## Fundamental to all applications:

| $f(x)$ | $f^{\prime}(x)$ |
| :---: | :---: |
| horiz. tangent | zero |
| increasing | positive |
| decreasing | negative |

### 9.9 HW Problem 8:

Rate of ascent for a balloon (in feet per minute) is given by

$$
a(t)=-0.3 t^{2}+3.9 t-2.928
$$



How will you answer these:
(a) Find the longest integral over which Balloon $A$ is rising.
(d) Find the time at which the balloon is rising the fastest?

## HW 9.9/1:

Given $g^{\prime}(x)=2 x^{2}-10 x+8$ $h^{\prime}(x)=6 x-16$


What does it mean when...
(a) ... $g^{\prime}(x)$ crosses the $x$-axis?
... $\mathrm{h}^{\prime}(\mathrm{x})$ crosses the x -axis?
(b) ... $\mathrm{g}^{\prime}(\mathrm{x})$ has a horizontal tangent (and how do you find it)?
(c) ... $\mathrm{h}^{\prime}(\mathrm{x})$ intersects $\mathrm{g}^{\prime}(\mathrm{x})$ ?

